Journal of Applied Mechanics and Technical Physics, Vol. 46, No. 2, pp. 275–280, 2005

SOME GEOMETRICALLY NONLINEAR PROBLEMS OF DEFORMATION OF INELASTIC PLATES AND SHALLOW SHELLS

I. Yu. Tsvelodub

UDC 539.37

This paper considers geometrically nonlinear problems of deformation of elastoplastic shallow shells and viscoelastoplastic plates where it is required to find kinematic loads for a given time interval such that a shell (plate) acquires prescribed residual deflections after these loads are applied and then removed. For some constraints, the correctness of the corresponding formulations (uniqueness of the solution and its continuous dependence on the problem data) is shown and iterative solution methods are justified.

Key words: *inverse problems, elastoplastic shallow shell, viscoelastoplastic plate, elastic "unbending," residual deflection.*

Papers [1–3] deal with the geometrically linear inverse problems of finding external loads that ensure the required residual deformation (i.e., residual deflections) of a viscoelastoplastic or elastoplastic plate within a specified time after unloading. In this paper, the results obtained in [1–3] are extended to the case of similar geometrically nonlinear problems where the deflections can far exceed the thickness of the plate (shallow shell) but remain much smaller than its dimensions in plan.

1. Constitutive Equations. We consider a shallow shell of variable (in the general case) thickness $h = h(x_1, x_2)$ whose middle surface is given by the equation $z = \Phi(x_1, x_2)$ in a chosen coordinate system Ox_1x_2z and is projected onto the plane Ox_1x_2 in a region S bounded by a closed contour γ.

The total strains of the shell are given by [4]

$$\varepsilon_{kl} = 0.5(u_{k,l} + u_{l,k}) - æ_{kl}w + 0.5w_{,k}w_{,l} - zw_{,kl},$$
$$æ_{kl} = -\Phi_{,kl}, \tag{1.1}$$

where u_k are the displacements in the plane Ox_1x_2 and w is the deflection which is allowed to be much greater than the thickness h but much smaller than the linear dimensions of the region S; the subscript k after comma denotes differentiation with respect to x_k. In (1.1) and below, we have $k, l = 1, 2$. [We note that relations (1.1) are valid in an arbitrary Cartesian system Ox_1x_2 that is not necessarily related to the principal curvatures of the shell.]

The equilibrium of have the form [4]

$$N_{kl,l} + X_k = 0, \qquad M_{kl,kl} + N_{kl}(w_{,kl} + æ_{kl}) + q = 0,$$

$$N_{kl} = \int\limits_{-h/2}^{h/2} \sigma_{kl}\, dz, \qquad M_{kl} = \int\limits_{-h/2}^{h/2} \sigma_{kl} z\, dz, \tag{1.2}$$

where σ_{kl}, N_{kl}, and M_{kl} are the stresses, membrane forces, and moments, respectively, q and X_k are the normal and tangential components of the external load, respectively, and summation from 1 to 2 is implied for repeated indices.

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; itsvel@hydro.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 46, No. 2, pp. 151–157, March–April, 2005. Original article submitted June 17, 2004.

Following [1–3], we assume that

$$\varepsilon_{kl} = a_{klmn}\sigma_{mn} + \varepsilon_{kl}^N, \tag{1.3}$$

where a_{klmn} and ε_{kl}^N are the components of the tensors of elastic compliances and inelastic (plastic or viscoplastic) strains, respectively.

The inverse problem of shell deformation can be generally formulated as follows [1, 2]: it is required to find external kinematic or force actions in the time interval $[0, t_*]$ such that after the external loads are applied and then removed the shell acquires a specified residual deflection $\tilde{w}_* = \tilde{w}_*(x_1, x_2)$, i.e., a specified residual shape (since $|\tilde{u}_{k*}| \ll |\tilde{w}_*|$) at the time $[0, t_*]$. At $t < 0$, the shell is in its natural state. The cases $t_* = 0$ and $t_* > 0$ correspond to the elastoplastic and viscoelastoplastic problems, respectively, which are considered below in a kinematic formulation under the condition of instantaneous elastic unloading at $t = t_*$.

At any time t $(0 \le t \le t_*)$, the deflection w can be written as (see [1–3])

$$w = w^e + \tilde{w}. \tag{1.4}$$

Here w^e is the elastic "unbending," i.e., the deflection which is a solution of the elastic problem for the current external loads $q = q(x_1, x_2, t)$ and $X_k = X_k(x_1, x_2, t)$ and the corresponding boundary conditions on γ, and $\tilde{w}$ is the current residual deflection after instantaneous removal of the indicated loads.

We assume that at the active deformation stage, i.e., before unloading, substantial inelastic strains ε_{kl}^N have been accumulated, so that the quantity w^e is small compared to the residual deflection $\tilde{w}$: $|w^e| \ll |\tilde{w}|$. Substituting (1.4) into (1.1) and ignoring the terms $w^e_{,k}w^e_{,l}$, we obtain [5]

$$\varepsilon_{kl} = 0.5(u_{k,l} + u_{l,k}) - æ_{kl}w + 0.5(\tilde{w}_{,k}w_{,l} + \tilde{w}_{,l}w_{,k} - \tilde{w}_{,k}\tilde{w}_{,l}) - zw_{,kl}. \tag{1.5}$$

Let us consider two states for which the residual deflections $\tilde{w}^{(i)}$ $(i = 1, 2)$ differ only slightly from $\tilde{w}$, i.e., $|\tilde{w} - \tilde{w}^{(i)}| \ll |\tilde{w}|$. In this case, the strains $\varepsilon_{kl}^{(i)}$ are determined by relations of the form of (1.5), in which u_k and w are replaced by $u_k^{(i)}$ and $w^{(i)}$, respectively, and the differences $\Delta\varepsilon_{kl} = \varepsilon_{kl}^{(1)} - \varepsilon_{kl}^{(2)}$ are given by

$$\Delta\varepsilon_{kl} = 0.5(\Delta u_{k,l} + \Delta u_{l,k}) - æ_{kl}\Delta w + 0.5(\tilde{w}_{,k}\Delta w_{,l} + \tilde{w}_{,l}\Delta w_{,k}) - z\Delta w_{,kl} \tag{1.6}$$

[the differences of the residual strains $\Delta\tilde{\varepsilon}_{kl}$ also satisfy relations (1.6), where Δw and Δu_k should be replaced by $\Delta\tilde{w}$ and $\Delta\tilde{u}_k$, respectively].

For the quantities $\Delta\varepsilon_{kl}$ from (1.6) and $\Delta\sigma_{kl}$ that satisfy the first two equations (1.2) (i.e., $\Delta N_{kl,l} + \Delta X_k = 0$), where $\Delta\varepsilon_{kl}$ and $\Delta\sigma_{kl}$ may not be related to one another, we evaluate the integral

$$I \equiv \frac{1}{2}\int\limits_S \int\limits_{-h/2}^{h/2} \Delta\varepsilon_{kl}\Delta\sigma_{kl}\, dz\, dx_1\, dx_2.$$

Performing calculations similar to those described in [5], for the geometrically nonlinear inverse relaxation problem of plate bending, we obtain

$$I = \frac{1}{2}\int\limits_\gamma \Big[(\Delta u_k + \tilde{w}_{,k}\Delta w)\Delta p_k + \Big(\Delta Q + \frac{\partial \Delta H}{\partial s}\Big)\Delta w - \Delta G\,\frac{\partial \Delta w}{\partial n}\Big]\, ds$$

$$+ \frac{1}{2}\int\limits_S [(\Delta u_k + \tilde{w}_{,k}\Delta w)\Delta X_k + \Delta w \Delta q_1]\, dx_1\, x_2, \tag{1.7}$$

$$-\Delta q_1 \equiv \Delta M_{kl,kl} + (\tilde{w}_{,kl} + æ_{kl})\Delta N_{kl}, \qquad \Delta p_k = \Delta N_{kl} n_l,$$

$$\Delta H = \Delta M_{kl} n_k t_l, \qquad \Delta G = \Delta M_{kl} n_k n_l, \qquad \Delta Q = \Delta M_{kl,l} n_k,$$

where n_k and t_k are the components of the unit normal and tangential vectors to the contour γ, respectively.

By virtue of the assumptions adopted above, the deflections $w^{(i)}$ differ only slightly from $\tilde{w}$, which implies that $N_{kl}^{(i)} w_{,kl}^{(i)} \approx N_{kl}^{(i)} \tilde{w}_{,kl}$. In this case, for the quantity Δq_1 defined in (1.7), from (1.2) we obtain

$$\Delta q_1 \approx \Delta q. \tag{1.8}$$

By analogy with [1, 2], for the specified deflection-increment field $\Delta w = \Delta w(x_1, x_2, t)$ with the known residual deflection $\tilde{w}$, we introduce the norm

$$\|\Delta w\|^2 = \frac{1}{2}\int\limits_S \int\limits_{-h/2}^{h/2} b_{klmn}\, \Delta\bar{\varepsilon}^e_{kl}\Delta\bar{\varepsilon}^e_{mn}\, dz\, dx_1\, dx_2 = \frac{1}{2}\int\limits_S \int\limits_{-h/2}^{h/2} \Delta\bar{\varepsilon}^e_{kl}\Delta\bar{\sigma}^e_{kl}\, dz\, dx_1\, dx_2, \tag{1.9}$$

where b_{klmn} are the components of the elastic-constant tensor inverse to a_{klmn}.

The quantities $\Delta\bar{\varepsilon}^e_{kl}$ are related to $\Delta\bar{u}^e_k$ by equations of the form of (1.6), where $\Delta\bar{u}^e_k$ is a solution of the elastic problem that includes the first two equations of (1.2) ($\Delta\bar{N}^e_{kl,l} + \Delta X_k = 0$) for specified loads ΔX_k, the equalities $\Delta\bar{\sigma}^e_{kl} = b_{klmn}\, \Delta\bar{\varepsilon}^e_{mn}$, and the corresponding conditions at γ (for $\Delta\bar{u}^e_k$ or $\Delta\bar{p}^e_k = \Delta\bar{N}^e_{kl} n_l$). If the loads ΔX_k are unknown, in addition to the quantities Δw and $\tilde{w}$, it is necessary to specify the functions $\Delta u_k = \Delta u_k(x_1, x_2, t)$. In this case, $\Delta\bar{u}^e_k = \Delta u_k$ and according to (1.6), we obtain $\Delta\bar{\varepsilon}^e_{kl} = \Delta\varepsilon_{kl}$.

One can see from (1.7) and (1.9) that $||\Delta w||^2$ is identical to the quantity I from (1.7), where the force characteristics of the external actions are denoted with an overbar and the superscript e and Δu_k is replaced by $\Delta\bar{u}^e_k$.

2. Elastoplastic Problem. We consider the case where the quantities ε^N_{kl} from (1.3) are plastic strains ε^p_{kl}. We assume that during active elastoplastic deformation, the shell deflection increases monotonically from zero to the sought quantity $w = w(x_1, x_2)$, which allows the deformation theory of plasticity to be used. The external loads are removed instantaneously, i.e., elastic unloading occurs. Thus, $t_* = 0$ and $\varepsilon^N_{kl} \equiv \varepsilon^p_{kl}$, and according to [3], we have

$$\varepsilon^p_{kl} = \begin{cases} 0, & \Sigma < \sigma_T, \\ \lambda_0\, \partial\Sigma/\partial\sigma_{kl}, & \Sigma \geq \sigma_T, \end{cases} \tag{2.1}$$

where $\Sigma = \Sigma(\sigma_{kl})$ is a homogeneous first-degree convex function, σ_T is the yield point, $\lambda_0 = \lambda_0(\Sigma) > 0$ is a specified function such that $\lambda_0'(\Sigma) > 0$ for a strain-hardening material, and λ_0 is the undetermined factor for an ideal plastic material (in this case, the second inequality in (2.1) is replaced by the equality $\Sigma = \sigma_T$).

It follows from (2.1) that for any two states in both the plastic or elastic regions, the following inequality holds:

$$\Delta\varepsilon^p_{kl}\, \Delta\sigma_{kl} \geq 0, \tag{2.2}$$

where the equality sign occurs in the cases discussed in [3].

The residual deflection $\tilde{w}$ from (1.4) corresponds to the strains $\tilde{\varepsilon}_{kl}$ defined by (1.1), where w and u_k are replaced by $\tilde{w}$ and $\tilde{u}_k$, respectively. In this case, the following relations are satisfied [1, 3]:

$$\varepsilon_{kl} - \tilde{\varepsilon}_{kl} = a_{klmn}\sigma^e_{mn}, \qquad \sigma^e_{mn} \equiv \sigma_{kl} - \rho_{kl}. \tag{2.3}$$

Here ρ_{kl} are the residual stresses in the shell after the removal of the external loads.

The problem is considered in the kinematic formulation: it is required to find a deflection $w = w(x_1, x_2)$ that ensures the specified residual deflection $\tilde{w} = \tilde{w}_*(x_1, x_2)$ after elastic unloading. The boundary conditions at γ are given by $w = \partial w/\partial n = 0$ (alternatively, one can specify other conditions that correspond to any of the four possible versions given for the case of small deflections of a plate in [1, 3]) and $u_k = 0$ (or $p_k \equiv N_{kl} n_l = 0$). We note that boundary conditions for the active loading stage may differ from those for unloading. If the external loads X_k are unknown, the displacements $u_k = u_k(x_1, x_2)$ need to be specified at the first stage (before unloading).

We assume that substantial plastic strains occur, i.e., $|w - \tilde{w}_*| \ll |\tilde{w}_*|$; therefore, the solution $w = w(x_1, x_2)$ is sought for in the neighborhood of the specified residual deflection $\tilde{w}_*(x_1, x_2)$.

It can be shown that if a solution of this problem exists, it is unique in the same sense as in [3] under the assumptions formulated above. Indeed, let $v = \{x \mid x = (x_1, x_2, z) \in R^3,\ (x_1, x_2) \in S,\ |z| \leq h/2\}$. Assuming that two solutions exist and using the symbol Δ to denote the corresponding differences, we evaluate the quantity

$$\tilde{I} \equiv \int\limits_v \Delta\tilde{\varepsilon}_{kl}\Delta\sigma_{kl}\, dv.$$

Taking into account relations (1.7) (where it is necessary to set $\tilde{w} = \tilde{w}_*$, $\Delta u_k = \Delta\tilde{u}_k$, and $\Delta w = \Delta\tilde{w}$) and (1.8) by virtue of the indicated boundary conditions on γ and using the fact that $\Delta\tilde{w} = 0$ everywhere in S, we find that $\tilde{I} = 0$. From this, taking into account (1.3) and (2.3) and using the equality

$$\int\limits_v a_{klmn}\,\Delta\sigma^e_{kl}\Delta\rho_{mn}\,dv=0 \tag{2.4}$$

which follows from (1.7) (for Δu^e_k and Δw^e), we obtain

$$\int\limits_v (a_{klmn}\,\Delta\rho_{kl}\,\Delta\rho_{mn}+\Delta\varepsilon^p_{kl}\,\Delta\sigma_{kl})\,dv=0.$$

By virtue of (2.2), this relation holds only if $\Delta\varepsilon^p_{kl}\,\Delta\sigma_{kl}=0$ and $\Delta\rho_{kl}=0$ everywhere in v, which implies [3] that the residual stresses ρ_{kl} in v, the plastic zone v_p, and the stresses σ_{kl} and σ^e_{kl} in v_p are determined uniquely. Therefore, in the region S_p that is the projection of v_p onto the plane Ox_1x_2, the deflection w is determined with accuracy up to a linear function of x_1 and x_2. If S_p is adjacent to the nonrectilinear part of the contour γ, on which the deflection w is specified, the deflection is determined uniquely in the region S_p. If the function $w=w(x_1,x_2)$ is analytic in S_p, it can be continued to the entire region S [3].

As in [3], this elastoplastic problem reduces to finding the deflection w from the functional equation

$$w=F(w),\qquad F(w)=w^e(w)+\tilde{w},$$

which can be solved by an iterative method:

$$w^{n+1}=F(w^n)=w^{en}+\tilde{w} \tag{2.5}$$

$[w^{en}=w^e(w^n),\ n=0,1,2,\ldots]$ using the zeroth approximation $w^0=\tilde{w}$.

Thus, in each iteration we have the direct problem of finding the elastic "unbending" $w^e=w^e(x_1,x_2)$ for the known function $w=w(x_1,x_2)$. This problem, which includes Eqs. (1.2) for specified loads X_k and q, the relations $\sigma^e_{kl}=b_{klmn}(\varepsilon_{mn}-\tilde{\varepsilon}_{mn})=b_{klmn}\varepsilon^e_{mn}$ implied by (2.3), where ε_{kl} and $\tilde{\varepsilon}_{kl}$ are given by (1.5), and the boundary conditions on γ given above, has a unique solution because the difference of the two possible solutions $\|\Delta w^e\|=0$, as follows from (1.7) by virtue of (1.8) (i.e., $\Delta q^e_1\approx\Delta q=0$) and zero conditions on γ.

Following [3] and taking into account (2.4), it is easy to show that the sequence (2.5) converges to the desired deflection $w=w(x_1,x_2)$ in the same sense as in [3].

3. Viscoelastoplastic Problem. Let the quantities ε^N_{kl} from (1.3) be the sum of plastic and viscous strains, whose rates depend on stresses and, possibly, on time:

$$\dot{\varepsilon}^N_{kl}=\dot{\varepsilon}^N_{kl}(\sigma_{mn},t).$$

We assume that these functions satisfy the condition [2]

$$\Delta\dot{\varepsilon}^N_{kl}\,\Delta\sigma_{kl}\geq\lambda a_{klmn}\,\Delta\sigma_{kl}\,\Delta\sigma_{mn},\qquad \lambda=\text{const},\quad \lambda>0. \tag{3.1}$$

We formulate a problem similar to that considered in [1, 2] for a plate in a kinematic, geometrically linear formulation. It is required to find a function $w_*=w_*(x_1,x_2)$ such that the residual deflection satisfies the condition $\tilde{w}(x_1,x_2,t_*)=\tilde{w}_*$ for the deflection $w=\varphi(t)w_*$ and displacements $u_k(x_1,x_2,t)=0$ $(0\leq t\leq t_*)$ at time $t=t_*$ after instantaneous removal of external loads $X_{k*}=X_k(x_1,x_2,t_*)$ and $q_*=q(x_1,x_2,t_*)$ and elastic unloading. Here $\tilde{w}_*=\tilde{w}_*(x_1,x_2)$ and $\varphi(t)$ are specified functions such that $\varphi(0)=0$ and $\varphi(t_*)=1$.

We confine our attention to a viscoelastoplastic plate assuming that $æ_{kl}=0$ in all formulas given above. By virtue of the above assumptions, for the differences between the strains corresponding to two states for which the residual deflections $\tilde{w}^{(i)}_*$ $(i=1,2)$ differ only slightly from $\tilde{w}_*$, we have relations of the form (1.6) for $t=t_*$, namely:

$$\Delta\varepsilon_{kl*}=0.5(\Delta u_{k*,l}+\Delta u_{l*,k}+\tilde{w}_{*,k}\Delta w_{*,l}+\tilde{w}_{*,l}\Delta w_{*,k})-z\Delta w_{*,kl}. \tag{3.2}$$

(The formulas for $\Delta\tilde{\varepsilon}_{kl*}$ are obtained from (3.2) by replacing Δu_{k*} and Δw_* by $\Delta\tilde{u}_{k*}$ and $\Delta\tilde{w}_*$, respectively.)

Since $w=\varphi(t)w_*$ and $u_k=0$ for $0\leq t\leq t_*$, from (1.1) and (3.2) we obtain

$$\Delta\varepsilon_{kl}=\varphi^2(t)\Delta\gamma_{kl}-z\varphi(t)\Delta w_{*,kl},\qquad \Delta\gamma_{kl}\equiv 0.5(\tilde{w}_{*,k}\Delta w_{*,l}+\tilde{w}_{*,l}\Delta w_{*,k}),$$

$$\Delta\varepsilon_{kl*}=\Delta\gamma_{kl}-z\Delta w_{*,kl},\qquad \Delta\dot{\varepsilon}_{kl}=\dot{\varphi}(t)[2\varphi(t)\Delta\gamma_{kl}-z\Delta w_{*,kl}].$$

It follows that

$$\|\Delta w_*\|^2 = I_1^2(\Delta\varepsilon_{kl*}) \equiv \frac{1}{2}\int\limits_v b_{klmn}\,\Delta\varepsilon_{kl*}\,\Delta\varepsilon_{mn*}\,dv = I_2^2 + I_3^2,$$

$$I_2^2 = \int\limits_S \frac{h}{2}\,b_{klmn}\,\Delta\gamma_{kl}\,\Delta\gamma_{mn}\,dS, \qquad I_3^2 = \int\limits_S \frac{h^3}{24}\,b_{klmn}\,\Delta w_{*,kl}\,\Delta w_{*,mn}\,dS, \tag{3.3}$$

$$\|\Delta\dot{w}\|^2 = I_1^2(\Delta\dot{\varepsilon}_{kl}) = [\dot{\varphi}(t)]^2[4\varphi^2(t)I_2^2 + I_3^2] \le 4[\dot{\varphi}(t)]^2\|\Delta w_*\|^2 \qquad (0 \le t \le t_*).$$

By analogy with [2], from (1.3) we obtain

$$2\dot{I}_4(\Delta\sigma_{kl})I_4(\Delta\sigma_{kl}) + I_5(t) = \int\limits_v \Delta\dot{\varepsilon}_{kl}\,\Delta\sigma_{kl}\,dv \le 2I_4(\Delta\sigma_{kl})I_1(\Delta\dot{\varepsilon}_{kl}),$$

$$I_4^2(\Delta\sigma_{kl}) = \frac{1}{2}\int\limits_v a_{klmn}\,\Delta\sigma_{kl}\,\Delta\sigma_{mn}\,dv,$$

$$I_5(t) = \int\limits_v \Delta\dot{\varepsilon}_{kl}^N(t)\Delta\sigma_{kl}(t)\,dv \qquad (0 \le t \le t_*).$$

With allowance for (3.1) and (3.3), these relations yield

$$\dot{I}_4 + \lambda I_4 \le 2|\dot{\varphi}|\,\|\Delta w_*\|, \qquad \text{i.e.,} \quad [I_4\exp(\lambda t)]^\bullet \le 2\|\Delta w_*\|\,|\dot{\varphi}|\exp(\lambda t).$$

Integrating this inequality with respect to time from zero to the current time t $(0 \le t \le t_*)$ and taking into account that $\Delta\sigma_{kl}|_{t=0} = 0$ [since $\varphi(0) = 0$], we obtain

$$I_4(\Delta\sigma_{kl}) \le \beta(t)\|\Delta w_*\|, \qquad \beta(t) = 2\exp(-\lambda t)\int\limits_0^t |\dot{\varphi}(t)|\exp(\lambda t)\,dt. \tag{3.4}$$

Since $\Delta\sigma_{kl} = \Delta\sigma_{kl}^e + \Delta\rho_{kl}$ and relations (1.7) (for $\tilde{w} = \tilde{w}_*$), (1.8), and (2.4) hold for $t = t_*$, we have [2]

$$I_4^2(\Delta\sigma_{kl*}) = I_4^2(\Delta\sigma_{kl*}^e) + I_4^2(\Delta\rho_{kl*}) \ge I_4^2(\Delta\sigma_{kl*}^e) = \|\Delta w_*^e\|^2.$$

Relation (3.4) yields

$$\|\Delta w_*^e\| \le \beta_*\|\Delta w_*\|, \qquad \beta_* = \beta(t_*). \tag{3.5}$$

Since $\|\Delta w_*\| = \|\Delta w_*^e + \Delta\tilde{w}_*\| \le \|\Delta w_*^e\| + \|\Delta\tilde{w}_*\|$, relation (3.5) implies the inequality $(1 - \beta_*)\|\Delta w_*\| \le \|\Delta\tilde{w}_*\|$, which ensures that for $\beta_* < 1$ the solution of the problem is unique and the operator $w_* = w_*(\tilde{w}_*)$ is continuous. The desired deflection w_* can be obtained as the limit of a sequence of form (2.5), i.e.,

$$w_*^{n+1} = w_*^e(w_*^n) + \tilde{w}_* \quad (n = 0, 1, 2, \ldots), \qquad w_*^0 = \tilde{w}_*,$$

since the problem reduces to solving the functional equation $w_* = F_1(w_*) \equiv w_*^e(w_*) + \tilde{w}_*$. We note that for $\beta_* < 1$, the operator F_1 is compressing [2].

We give some examples of the function $\varphi = \varphi(t)$ provided that $\beta_* < 1$. From (3.4) for $\varphi(t) = t/t_*$, we find that $\beta_* = 2[1 - \exp(-\gamma_*)]/\gamma_*$, where $\gamma_* = \lambda t_*$; therefore, $\beta_* < 1$ for $\gamma_* \ge 2$. In a more general case, using the Cauchy–Bunyakowsky inequality, from (3.4) we obtain

$$\beta_* \le 2æ_*\exp(-\gamma_*)\Big(\int\limits_0^{t_*}\exp(2\lambda t)\,dt\Big)^{1/2} = \sqrt{2}\,æ_*\Big[\frac{1 - \exp(-2\gamma_*)}{\lambda}\Big]^{1/2} < \frac{\sqrt{2}\,æ_*}{\sqrt{\lambda}},$$

$$æ_*^2 = \int\limits_0^{t_*}(\dot{\varphi})^2\,dt.$$

If, for example, $\varphi(t) = (t/t_*)^\alpha$ ($\alpha > 1/2$), then $æ_* = \alpha/\sqrt{(2\alpha - 1)t_*}$ and to satisfy the condition $\beta_* < 1$, it is sufficient that $\gamma_*(1 - \sqrt{1 - 2\gamma_*^{-1}}\,) \le 2\alpha \le \gamma_*(1 + \sqrt{1 - 2\gamma_*^{-1}}\,)$, which is possible for $\gamma_* \ge 2$.

We note that in the inequality of the form of (3.4) obtained in [2] for a similar geometrically linear problem, $\beta(t)$ is smaller than that in (3.4) by a factor of 2. Therefore, the corresponding constraints on the function $\varphi = \varphi(t)$ are weaker: for example, the inequality $\beta_* < 1$ holds for any function $\varphi = \varphi(t)$ which increases monotonically from zero to unity ($0 \le t \le t_*$).

We also note that as in [2], the minimum value of β_* from (3.5) corresponds to the relaxation mode of deformation where $\varphi(0) = 0$, $\dot{\varphi} > 0$ ($0 < t < t_0$), and $\varphi = 1$ ($t_0 \le t \le t_*$) as $t_0 \to 0$; in this case, we have $\beta_* = 2\exp(-\gamma_*)$. Consequently, the condition $\beta_* < 1$ is possible only if $\gamma_* > \ln 2 \approx 0.693$.

This work was supported by the Russian Foundation for Basic Research (Grant No. 02-01-00643) and the Foundation “Leading Scientific Schools of Russia” (Grant No. NSh-319.2003.1).

REFERENCES

1. I. Yu. Tsvelodub, “Inverse deformation problems of inelastic plates,” *Izv. Ross. Akad. Nauk. Mekh. Tverd. Tela*, No. 1, 96–106 (1996).
2. I. A. Banshchikova and I. Yu. Tsvelodub, “One class of inverse shape-forming problems of viscoelastic plates,” *J. Appl. Mech. Tech. Phys.*, **37**, No. 6, 122–131 (1996).
3. I. Yu. Tsvelodub, “Inverse elastoplastic problem of a plate,” *J. Appl. Mech. Tech. Phys.*, **40**, No. 4, 186–194 (1999).
4. A. S. Vol’mir, *Flexible Plates and Shells* [in Russian], Gostekhteoretizdat, Moscow (1956).
5. I. Yu. Tsvelodub, “Some inverse creep problems of plate bending,” *Izv. Akad. Nauk. SSSR, Mekh. Tverd. Tela*, No. 5, 126–134 (1985).